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Decode-and-Forward Buffer-Aided Relay Selection in Cognitive Relay Networks

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***Abstract*—This paper investigates decode-and-forward (DF) buffer-aided relay selection for underlay cognitive relay networks in the presence of both primary transmitter and receiver. We propose a novel buffer aided relay selection scheme for the cognitive relay network, where the best relay is selected with the highest signal-to-interference-ratio (SIR) among all available source-to-relay and relay-to-destination links while keeping the interference to the primary destination within a certain level. A new closed-form expression for the outage probability of the proposed relay selection scheme is obtained. Both simulation and theoretical results are shown to confirm performance advantage over the conventional max-min relay selection scheme, making the proposed scheme attractive for cognitive relay networks.**

***Index Terms*—Cognitive relay networks, relay selection, buffer-aided decode-and-forward relay**

I. Problem Statement

RELAY selection provides an efficient way to harvest the diversity gain in a cognitive relay network (CRN). When only the best relay is selected for transmission, not only the system complexity but also the interference to the primary user is significantly lower than that when all relays participate in transmission. A typical relay selection system in a CRN is shown in Fig. 1, where there is one primary source (PS), one primary destination (PD), one secondary source node (SS), one secondary destination node (SD) and a number of relays $SR_k, k \in (1, 2, ..., K)$. The channel coefficients and gains for the channel "$a \to b$" are labelled respectively as h_{ab} and $\gamma_{ab} = |h_{ab}|^2$.

As is shown in Fig. 1 (a), if the relay SR_k is selected to receive data from the secondary source SS, due to the interference from the primary source PS, the received signal at SR_k is given by

$$\mathbf{y}_{sr_k} = \sqrt{P_{ss}} h_{sr_k} \mathbf{s} + h_{pr_k} \sqrt{P_{ps}} \mathbf{s}^{'} + \mathbf{n}_{r_k}, \tag{1}$$

where $\mathbf{s}$ and $\mathbf{s}^{'}$ are transmission signal vectors from SS and PS respectively, P_a represents the transmission power at the node a ($a \in \{ss, sr_k, ps\}$) and $\mathbf{n}_{r_k}$ is the noise vector at SR_k. On the other hand, as is shown in Fig. 1 (b), if the relay SR_k is selected to forward the data to the secondary destination SD, the received signal at SD is given by

$$\mathbf{y}_{r_k d} = \sqrt{P_{sr_k}} h_{r_k d} \mathbf{s} + h_{pd} \sqrt{P_{ps}} \mathbf{s}^{'} + \mathbf{n}_d, \tag{2}$$

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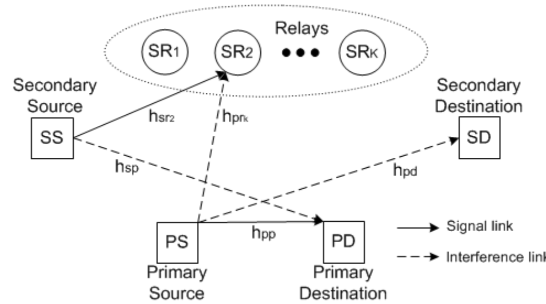
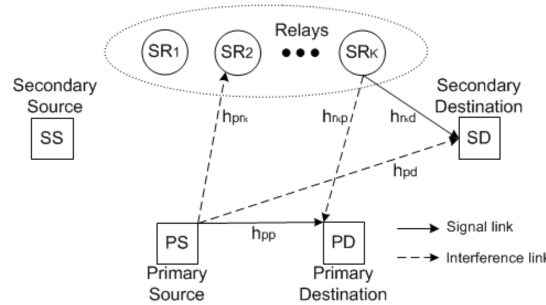
where $\mathbf{n}_d$ is the noise vector at SD.



(a) Source to relay transmission



(b) Relay to destination transmission

Fig. 1. Relay selection in the cognitive relay network.

In the underlay cognitive system, the secondary transmission nodes including SS and SR_k are only allowed to share the spectrum with the primary user PD if the corresponding interfering power to PD is below a pre-defined level I_{th} such that $P_{ss}\gamma_{sp} \leq I_{th}$ and $P_{sr_k}\gamma_{r_k p} \leq I_{th}$. With these power constraints, the received signal-to-interference (SIR) at the selected relay SR_k and the destination D become

$$\begin{aligned} \mathrm{SIR}_{sr_k} &= \frac{P_{ss}|h_{sr_k}|^2}{P_{ps}|h_{pr_k}|^2} = \frac{I_{th}\gamma_{sr_k}}{\gamma_{sp}\gamma_{pr_k}}, \\ \mathrm{SIR}_{r_k d} &= \frac{P_{sr_k}|h_{r_k d}|^2}{P_{ps}|h_{pd}|^2} = \frac{I_{th}\gamma_{r_k d}}{\gamma_{r_k p}\gamma_{pd}}, \end{aligned} \tag{3}$$

respectively, where the transmission power at the primary source P_{ps} is normalized to unit without losing generality.

The objective of the relay selection in the CRN is to choose the "best" relay node such that the corresponding end-to-end capacity from SS to SD is maximized, subject to the constraint that the interferences at the primary destination PD are below a certain level. Of particular interest in the CRN

$$R_{best,max-min} = \arg\max_{SR_k} \left\{ \min\left\{ \frac{I_{th}\gamma_{sr_1}}{\gamma_{sp}\gamma_{pr_1}}, \frac{I_{th}\gamma_{r_1 d}}{\gamma_{r_1 p}\gamma_{pd}} \right\}, \cdots, \min\left\{ \frac{I_{th}\gamma_{sr_K}}{\gamma_{sp}\gamma_{pr_K}}, \frac{I_{th}\gamma_{r_K d}}{\gamma_{r_K p}\gamma_{pd}} \right\} \right\}. \quad (4)$$

is the interference-limited scenario wherein the interference power from the primary source is dominant relative to the noise so that the noise effects can be ignored [1], and then the capacity is mainly dependent on the SIR. It is known that, if the relay node applies decode-and-forward (DF), the corresponding end-to-end capacity is the minimum of those for the source-to-relay and relay-to-destination links. Therefore, the traditional max-min relay selection can be generalized for the relay selection in the CRN, where the best relay node is selected with the maximum SIR calculated as the $\min\{\mathrm{SIR}_{sr_k}, \mathrm{SIR}_{r_k d}\}$ from all available relay nodes. Or from (3) the max-min scheme chooses the best relay node for the CRN as (4) in the top of this page. As is shown in (3), because the transmission power from the source to every relay SR_k is limited according to the same interference constraint at the primary destination PD, there exists a common term "γ_{sp}" in every SIR_{sr_k}. Similarly, because every $SR_k \to SD$ transmission suffers from the same interference from the primary source PS, there also exists a common term "γ_{pd}" in every $\mathrm{SIR}_{r_k d}$. These two common terms imply that the "min" terms within the "max" operation in (4) become correlated. This translates into the fact that the best relay is selected among dependent "candidates", or the full diversity cannot be achieved even when all relevant channel coefficients are independently and identically distributed (i.i.d.). This is very different from the conventional relay selection where the best relay is usually selected amongst independent candidates. The correlation among selection candidates is thus the key issue in the CRN relay selection scheme considered in this work.

Relay selection in the CRN has attracted much attention recently. In [2], a *max-min* based relay selection similar to (4) was proposed in the CRN, though only the primary destination PD is available (no primary source PS) in the system. Similar to (4), the candidates for the relay selection in [2] are also correlated. However, the outage analysis in [2] assumed that there exist multiple independent links between the secondary source and primary destination so that the candidates for relay selection become independent. This is not correct because there is only one secondary source and primary destination in the system respectively, or there are no multiple secondary source to primary destination links. Some earlier works (e.g. [3]) related to such CRNs also failed to consider the correlation in the relay selection. The correlation in the cognitive relay selection was identified in [4], and a "half" DF relay selection scheme was proposed where in the first phase the source broadcasts data to all relays and only in the second phase applies the relay selection so that only the selected relay is used for data transmission. A similar relay selection approach was also considered in [5] so that the outage performance can be analyzed. While the "half" relay selection successfully avoids the correlation issue in the CRN relay selection, it is at the price of losing efficiency because all relays (rather than only the selected relay) are involved in transmission in the first phase. Alternatively, the correlation in the CRN relay selection may also be avoided by assuming the link between the secondary and primary sources is constant, but this only applies to some particular systems such as when the secondary source and primary user have little mobility [6].

Most current CRN relay selection approaches (including the aforementioned) assume there is no primary source for simplicity. In practice, both primary transmitter and receiver may present, in which the interference from the primary transmitter to the secondary users cannot be ignored [1], [7]. This motivates us to investigate relay selection in the more general CR network as is shown in Fig. 1. On the other hand, it is recently recognized that the performance of conventional relay selection can be further improved by relaxing the constraint that the best source-to-relay and relay-to-destination links for a packet transmission must be determined altogether. This is achieved by introducing data buffers at the relay nodes (e.g. [8], [9]). Of particular interest is the buffer-aided max-link relay selection where the best link is always selected among all available source-to-relay and relay-to-destination links [8]. In this paper, we propose the max-ratio relay selection for the CRN, where the selected relay achieves the highest signal-to-interference-ratio (SIR) at the secondary destination while satisfying the interference constraint at the primary receivers. The main contributions of this paper are listed as follows:

- Proposing the buffer-aided max-ratio relay selection in the underlay cognitive relay network. Because the best source-to-relay and relay-to-destination links are selected separately, the proposed scheme provides a more efficient way to handle the correlation among relay selection candidates than existing approaches, a key issue in the CRN relay selection. As far as the authors are aware, this is also the first relay selection in the CRN with both primary transmitter and receiver available.
- Deriving the closed-form expression of the outage probability for the proposed relay selection scheme. Because both the primary transmitter and receiver are present, the analysis is much more involved than those for both the conventional and the existing cognitive relay selection schemes. The analysis not only provides a deep insight in understanding the proposed scheme but also shows a potential approach to analyze similar systems in the future.

II. Max-ratio Relay Selection

In the buffer aided relay selection, each relay is equipped with a data buffer Q_k ($1 \le k \le K$) of finite size L (in the number of data packets), and the data packets in the buffer follow the "first-in-first-out" rule. The secondary transmission related channels can be divided into three groups: *secondary transmission channels* h_{sr_k} and $h_{r_k d}$ for $SS \to SR_k$ and $SR_k \to SD$ respectively, *secondary interfering channels* h_{pr_k}

and h_{pd} for $PS \rightarrow SR_k$ and $PS \rightarrow SD$ respectively, and *primary interfering channels* h_{sp} and h_{r_kp} for $SS \rightarrow PD$ and $SR_k \rightarrow PD$ respectively. We assume all channels are quasi-static Rayleigh fading so that the channel coefficients remain unchanged during one packet duration but independently vary from one packet time to another. We also assume that channels within every group are i.i.d. fading, but channels for different groups may have different average gains, or we have $\lambda_{sr_k} = \lambda_{r_kd}$, $\lambda_{pr_k} = \lambda_{pd}$ and $\lambda_{sp} = \lambda_{r_kp}$ for all k. This is a more practical assumption than those in many existing approaches where all channels are assumed to be i.i.d. fading (e.g. [5], [8], [10]). We also assume that the secondary source SS and relays SR_k have the channel-state-information (CSI) knowledge from themselves to the primary destination respectively, so that the transmission powers at SS and SR_k can be determined. Moreover, we assume that the secondary destination node has global CSI[1] and buffer state information for all relays, and selects a relay for transmission through an error-free feedback channel [13].

In the max-ratio relay selection, at any time, the best transmission link with the highest SIR is selected among all available source-to-relay and relay-to-destination links. A source-to-relay or a relay-to-destination link is considered available when the buffer of the corresponding relay node is not full or empty respectively. To be specific, if a source-to-relay link is selected, the source node transmits one data packet to the corresponding relay node. If the selected relay can successfully decode the data, the decoded packet is stored in the buffer and the number of data packets in the buffer is increased by one. On the other hand, if a relay-to-source link is selected, the corresponding relay transmits the earliest stored packet in the buffer to the destination. If the destination can successfully decode the packet, the number of packets in the buffer is decreased by one.

The best selected relay (either for transmission or reception) in the max-ratio scheme can be obtained as $R_{best,max-ratio} = \arg \max\limits_{SR_k}\{(\mathrm{SIR}_{sr_k},\ \mathrm{SIR}_{r_kd}) \mid \text{for all available links}\}$. Because the source-to-relay and relay-to-destination links are determined separately, from (3), the max-ratio relay selection rule can be expressed as (5) in the top of the next page, where $\Psi(Q_k)$ gives the number of data packets in the buffer Q_k.

The outage probability can be defined as the probability that the selected link is in outage as

$$P_{out} \triangleq \begin{cases} \mathbb{P}\{(1/2)\log_2(1+\mathrm{SIR}_{sr_k}) < C_{th}\} & \text{R reception,} \\ \mathbb{P}\{(1/2)\log_2(1+\mathrm{SIR}_{r_kd}) < C_{th}\} & \text{D reception,} \end{cases} \tag{6}$$

where C_{th} is the target rate, and the factor 1/2 captures the fact that it takes two time slots to transmit any packet from the source to the destination.

III. Outage Probability Analysis

This section analyzes the outage probability of the max-ratio relay selection in the CRN. At any time, the numbers of data packets in every buffer form a "state". Because there are K available relays and every relay is equipped with a buffer of size L, there are $(L+1)^K$ states in total. The l-th state vector is defined as

$$\mathbf{s}_l = [\Psi_l(Q_1), \cdots, \Psi_l(Q_K)]^{\mathrm{T}}, \qquad l = 1, \cdots, (L+1)^K \tag{7}$$

where $\Psi_l(Q_k)$ gives the number of data packets in buffer Q_k at state s_l. It is clear that $0 \le \Psi_l(Q_k) \le L$.

We assume that state s_l corresponds to the pair of $(K_{1,l}, K_{2,l})$, where $K_{1,l}$ and $K_{2,l}$ are the numbers of available links for source-to-relay and relay-to-destination transmission at state s_l respectively. By considering all possible available available links for $K_{1,l}$ and $K_{2,l}$, the outage probability of the overall system can be obtained as

$$P_{out} = \sum_{l=1}^{(L+1)^K} \pi_l \overline{p}_{s_l}^{(K_{1,l},K_{2,l})}, \tag{8}$$

where $\overline{p}_{s_l}^{(K_{1,l},K_{2,l})}$ is the outage probability when the state is at s_l, and π_l is the stationary probability for the state s_l. The following two sub-sections show the calculation of $\overline{p}_{s_l}^{(K_{1,l},K_{2,l})}$ and $\boldsymbol{\pi}_l$ respectively.

A. $\overline{p}_{s_l}^{(K_{1,l},K_{2,l})}$: *outage probability for state* s_l

Separating the common terms γ_{sp} and γ_{pd} in the max-ratio relay selection rule in (5) gives (9) in the top of the next page. For the state s_l, there are $K_{1,l}$ and $K_{2,l}$ terms in the first and second part maximization within the "outer" max operation in (9) respectively. For clear expression, we let $w_k = \frac{I_{th}\gamma_{sr_k}}{\gamma_{pr_k}}$, $w = \max\{w_k\}$ and $x = \frac{w}{\gamma_{sp}}$, corresponding to the relay selection from the source-to-relay links. Similarly, we let $v_k = \frac{I_{th}\gamma_{r_kd}}{\gamma_{r_kp}}$, $v = \max\{v_k\}$ and $y = \frac{v}{\gamma_{pd}}$ for the relay-to-destination selection. And finally we let $z = \max\{x, y\}$ to complete the max-ratio relay selection for the overall system. It is clear that z gives the instantaneous SIR of the selected link. Thus the outage probability corresponding to $(K_{1,l}, K_{2,l})$ for the target rate C_{th} is given by

$$\overline{p}_{s_l}^{(K_{1,l},K_{2,l})} = P(z < C_{th}) = F_Z(z = S_{th}), \tag{10}$$

where $F_Z(z)$ is the cumulative distribution function (CDF) of z and $S_{th} = 2^{2C_{th}} - 1$ which is the target SNR. The CDF of z is derived as below.

First, for exponentially distributed channel gains, the CDF of $w_k = \frac{I_{th}\gamma_{sr_k}}{\gamma_{pr_k}}$ can be obtained as $F_{W_k}(w_k) = \frac{w_k}{L_1+w_k}$, where $L_1 = \frac{I_{th}\lambda_{sr_k}}{\lambda_{pr_k}}$ and $\lambda_{ab} = \mathrm{E}|\gamma_{ab}|^2$ representing the average channel gain for channel h_{ab}. Because the common term γ_{sp} is taken out of all w_k and all channels are assumed to be i.i.d, all of w_k are also i.i.d.. Further recalling that there are $K_{1,l}$ source-to-relay links available, the CDF of $w = \max\{w_k\}$ is given by

$$F_W(w) = (F_{W_k}(w))^{K_{1,l}} = \left(\frac{x}{L_1+x}\right)^{K_{1,l}}. \tag{11}$$

Because w and γ_{sp} are independent, the CDF of $x = w/\gamma_{sp}$ is obtained as

$$F_X(x) = \int_0^{\infty} \left(\frac{x\gamma_{sp}}{L_1+x\gamma_{sp}}\right)^{K_{1,l}} \frac{1}{\lambda_{sp}} e^{-\frac{\gamma_{sp}}{\lambda_{sp}}} d\gamma_{sp}. \tag{12}$$

[1]The CSI is usually estimated through pilots and feedback (e.g. [11]), and the CSI estimation without feedback may also be applied (e.g [12]). Further detail of the CSI estimation is beyond the scope of this paper.

$$R_{best,max-ratio} = \arg\max_{SR_k} \left\{ \max_{SR_k:\Psi(Q_k)\neq L} \left\{ \frac{I_{th}\gamma_{sr_k}}{\gamma_{sp}\gamma_{pr_k}} \right\}, \quad \max_{SR_k:\Psi(Q_k)\neq 0} \left\{ \frac{I_{th}\gamma_{r_k d}}{\gamma_{pd}\gamma_{r_k p}} \right\} \right\}, \tag{5}$$

$$R_{best,max-ratio} = \arg\max_{SR_k} \left\{ \frac{\max\limits_{SR_k:\Psi(Q_k)\neq L} \left\{ \frac{I_{th}\gamma_{sr_k}}{\gamma_{pr_k}} \right\}}{\gamma_{sp}}, \quad \frac{\max\limits_{SR_k:\Psi(Q_k)\neq 0} \left\{ \frac{I_{th}\gamma_{r_k d}}{\gamma_{r_k p}} \right\}}{\gamma_{pd}} \right\}. \tag{9}$$

The diversity gain from the source-to-relay selection is clearly reflected in (11). In fact, because the common factor γ_{sp} can be separated out, $\max\{w_k\}$ and $\max\{\frac{w_k}{\gamma_{sp}}\}$ lead to the same selected link, or they correspond to similar diversity gain. This is very different from the traditional max-min relay selection as is shown in (4), where the common factors γ_{sp} and γ_{pd} cannot be separated.

From (12), the closed-form expression of the CDF of x can be obtained as (13) in the top of the next page, where $\mathrm{Ei}(1,a) = \int_1^\infty \frac{\exp(-ta)}{a}dt, a > 0$, $\Gamma(\bullet)$ is the gamma function, and $\mathcal{MG}([[\ ],[\ ]],[[\bullet,\bullet],[\ ]],\bullet)$ is the Meijer G function [14].

We may define the diversity order from the source-to-relay selection as

$$d_{sr}^{(K_{1,l})} = -\lim_{\lambda\to\infty} \frac{\log(F_X(x))}{\lambda}, \tag{14}$$

where λ is the average channel gain. Numerical verification based on (13) reveals that the diversity order from the source-to-relay relay selection is close to $K_{1,l}$. This will be verified in the simulation section.

On the other hand, the CDF of y for the relay-to-destination selection can be obtained similarly to (13), and the diversity order for the relay-to-destination selection $d_{rs}^{(K_{2,l})}$ is also shown close to the number of the available relay-to-destination links $K_{2,l}$.

Finally, because the source-to-relay selection x and relay-to-destination selection y are independent, the CDF of $z = \max(x, y)$ is obtained as $F_Z(z) = F_X(z)F_Y(z)$. And the overall diversity order when the buffer state is at s_l (with the pair $(K_{1,l}, K_{2,l})$) is given by

$$d^{(K_{1,l},K_{2,l})} = d_{sr}^{(K_{1,l})} + d_{rd}^{(K_{2,l})}. \tag{15}$$

B. The stationary distribution probability π_l

The Markov chain can be used to model the transitions between the buffers states. Suppose at time t, the state is at s_n. At time $t+1$, if the received data can be successfully decoded, there must be one relay either receiving or transmitting a data packet, so that the number of packets in the corresponding buffer is increased or decreased by one respectively. Depending on which relay receives or transmits data, at time $t+1$, the buffers may move from state s_l to several possible states. We denote U_l as the set containing all states which can be moved from s_l.

Because the channels within secondary transmission, secondary interfering and primary interfering channels are i.i.d. fading, it is clear from (3) that the SIR-s for all channels are i.i.d. so that the probability to select any link is equable as $1/(K_{1,l} + K_{2,l})$. Further noting that the state remains unchanged if outage occurs (or the decoding is not successful), the probabilities that the state s_l moves to a state in U_l is given by

$$p_{s_l} = \frac{1 - \overline{p}_{s_l}^{(K_{1,l},K_{2,l})}}{K_{1,l} + K_{2,l}}. \tag{16}$$

We denote $\mathbf{A}$ as the $(L+1)^K \times (L+1)^K$ state transition matrix, where the entry $\mathbf{A}_{n,l} = P(X_{t+1} = s_n | X_t = s_l)$ which is the transition probability to move from state s_l at time t to state s_n at time $(t+1)$. With the above analysis, we have

$$\mathbf{A}_{n,l} = \begin{cases} \overline{p}_{s_l}^{(K_{1,l},K_{2,l})}, & \text{if} \quad s_n = s_l, \\ p_{s_l}, & \text{if} \quad s_n \in U_l, \\ 0, & \text{elsewhere}, \end{cases} \tag{17}$$

Because the transition matrix $\mathbf{A}$ is column stochastic, irreducible and aperiodic[2], the stationary state probability vector is obtained as (see [8], [16], [17])

$$\boldsymbol{\pi} = (\mathbf{A} - \mathbf{I} + \mathbf{B})^{-1}\mathbf{b}, \tag{18}$$

where $\boldsymbol{\pi} = [\pi_1, \cdots, \pi_{(L+1)^K}]^{\mathrm{T}}$, $\mathbf{b} = (1, 1, ..., 1)^T$, $\mathbf{I}$ is identity matrix and $\mathbf{B}_{n,l} = 1, \forall n, l$.

C. Discussion

Substituting (10) and (18) into (8) gives the outage probability of the max-ratio scheme. Or the outage probability can be expressed in the matix/vector form as

$$P_{out} = \mathrm{diag}(\mathbf{A})\boldsymbol{\pi}, \tag{19}$$

where $\mathrm{diag}(\mathbf{A})$ is the vector consisting of the diagonal elements of $\mathbf{A}$.

The overall outage probability is the "average" of the outage probability $\overline{p}_{s_l}^{(K_{1,l},K_{2,l})}$ over all possible $(K_{1,l}, K_{2,l})$. The distribution of $(K_{1,l}, K_{2,l})$ depends on both the number of relays K and the relay buffer size L. Particularly, when the buffer size $L = 1$, we always have $K_{1,l} + K_{2,l} = K$. As a result, the diversity order from the overall scheme is close to K.

In another extreme, if the relay buffer size $L \to \infty$, similar to that in [8], it can be shown that probabilities for $K_{1,l} = K$ and $K_{2,l} = K$ are one, or we have

$$\lim_{L\to\infty} P_{out} = \overline{p}_{s_l}^{(K_{1,l}=K,K_{2,l}=K)}. \tag{20}$$

[2] *Column stochastic* means all entries in any column sum up to one, *irreducible* means that it is possible to move from any state to any state, and *aperiodic* means that it is possible to return to the same state at any steps [15], [16].

$$F_X(x) = \begin{cases} 1, & \text{if} \quad K_{1,l} = 0, \\ 1 - \frac{L_1}{\lambda_{sp}x} e^{\frac{L_1}{\lambda_{sp}x}} \mathrm{Ei}(1, \frac{L_1}{\lambda_{sp}x}), & \text{if} \quad K_{1,l} = 1, \\ \left(\frac{\lambda_{sp}x}{L_1}\right)^{K_{1,l}-1} \frac{\mathcal{MG}\left([[0],[\ ]],[[K_{1,l}-1, K_{1,l}],[\ ]], \frac{L_1}{\lambda_{sp}x}\right)}{\Gamma(K_{1,l})}, & \text{elsewhere.} \end{cases} \tag{13}$$

Correspondingly, the diversity order is close to $2K$. In general, the diversity order of the max-ratio scheme is between K and $2K$.

In the max-ratio relay selection, different packets may have different delays because a packet can only be transmitted if the corresponding link is selected. Of particularly interest is the average packet delay which includes the delays at both the source and relay nodes. At the source node, because we assume every channel within a group are i.i.d., the probability that the source is selected for transmission is 1/2. In comparison, in the traditional relay selection scheme without any relay buffers, because the source node always transmits at the odd time slots and waits at the even time slots, the probability that the source transmits at any time is also 1/2. Therefore, the average delay at the source node is the same for the max-ratio and traditional schemes. Since the delay at the source in the traditional relay selection is 1, the source delay in the max-link scheme is also $D_{ave}^{(Source)} = 1$.

On the other hand, the average packet delay at the relay node in the max-ratio scheme can be obtained using Little's law [18]: *the average delay multiplying the throughput gives the average queuing length*. In the max-ratio scheme, because it takes two time slots to deliver a packet (though the two time slots may not be consecutive), the overall throughput of the whole system is 1/2. Because all channels within a group are i.i.d., the probability for a packet transmission via any of the relays is the same. Therefore, the throughput at any relay is $1/(2K)$. Furthermore, from the Markov model of the relay buffer, the average number of packets (queuing length) in a relay buffer can be obtained as $\sum_{l=1}^{(L+1)^K} \pi_l \Psi_l(Q_k)$. Then from Little's law, the average delay at the relay is given by:

$$D_{ave}^{(Relay)} = \frac{1}{(1/2)/K} \sum_{l=1}^{(L+1)^K} \pi_l \Psi_l(Q_k) = KL. \tag{21}$$

Combining the delay at the source and the relay then gives the overall average delay in the max-ratio system as

$$D_{ave} = D_{ave}^{(Source)} + D_{ave}^{(Relay)} = 1 + KL. \tag{22}$$

IV. Numerical Simulations

In the simulations below, the pre-defined level $I_{th} = 1$, and the average channel gains are set as $\lambda_{sp} = \lambda_{r_kp} = 10$ dB and $\lambda_{pr_k} = \lambda_{pd} = 10$ dB. The transmission power of primary transmitter and channel noise are normalized to unit.

Fig. 2 verifies the theoretical analysis for the proposed max-SIR-link scheme with simulations. We have performed extensive simulations with different numbers of relays and buffer sizes. While all simulation results match the theoretical analysis, only a few are shown in Fig. 2 for better illustration. It is clearly shown that the outage probability decreases as the number of relays and buffer size increases. For example, for target rate $C_{th} = 0.5$ bits per channel use (BPCU), when the number of relays and buffers (K, L) increases from (2, 2) to (5, 5), the outage probability drops by approximately 40 dB. This is because that higher diversity is obtained with more relays and higher coding gain is obtained with larger buffer size.

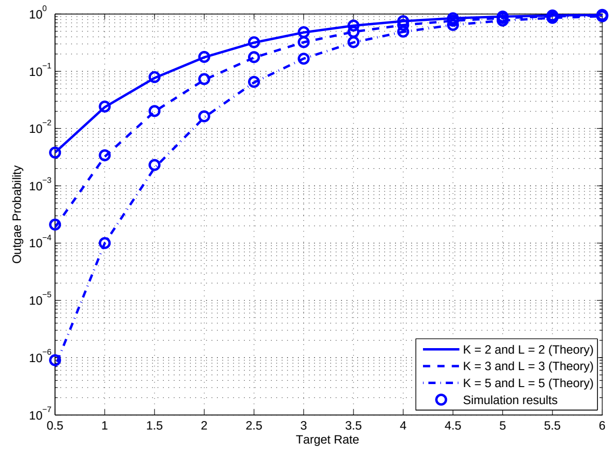


Fig. 2. Theoretical and simulation outage probability *vs* target rate for the proposed max-SIR-link relay selection, where $\lambda_{sr_k} = \lambda_{r_kd} = 30$ dB.

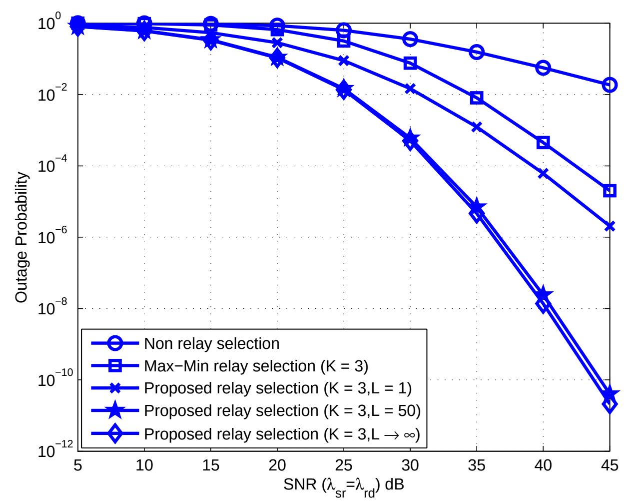


Fig. 3. The comparison of outage probability for different relay selection policies with $K = 3$ relays, L = 1, 50 and ∞ versus the different channel SNR, where $C_{th} = 1$ BPCU.

Fig. 3 compares the outage probabilities of the proposed max-SIR-link, conventional max-min and no relay selection schemes, where the number of relays is set as $K = 3$, the relay buffer sizes for the proposed approach are set as $L = 1, 50, \infty$

respectively, and for better illustration only theoretical results for the proposed scheme are shown. It is clearly shown that, even with buffer size $L = 1$, the proposed relay selection still has better outage performance than the conventional max-min scheme. This is due to the coding gain from the max-ratio approach. Particularly, for the proposed max-ratio selection, it is clearly shown that the diversity order is close to $K = 3$ for $L = 1$ and close to $2K = 6$ for $L \to \infty$. This matches well the analysis in Section III. Fig. 3 also shows that, for the proposed approach, the outage performance improves with larger buffer size, but the improvement becomes less significant when the buffer size is large enough. In particular, with $L = 50$, the outage performance is almost the same as that for $L \to \infty$.

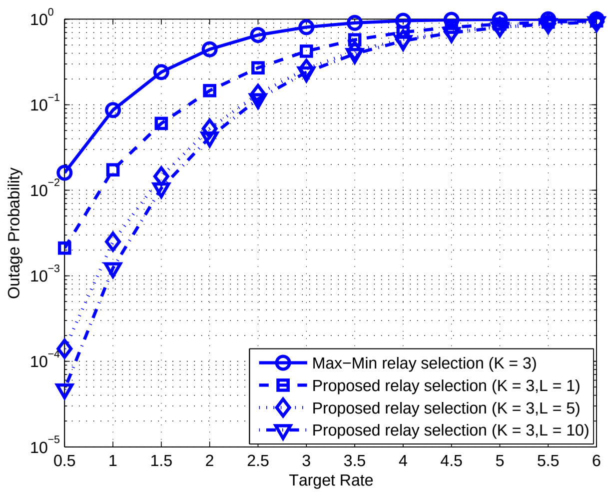


Fig. 4. The comparison of outage probability for different relay selection policies with $K = 3$ relays, $L = 1$, 5 and 10 versus the different target rate in n.i.d. system.

Fig. 4 compares the outage probabilities of the proposed max-SIR-link and conventional max-min in n.i.d. system, where the number of relays is set as $K = 3$, the relay buffer sizes for the proposed approach are set as $L = 1$, 5 and 10, respectively. And all of channels from SS to SR and form SR to SD are randomly selected from 25 to 35 dB, because of relay locations and path loss. The outage probability performance of our proposed scheme is better than that of max-min scheme.

V. Conclusions

This paper proposed the DF buffer-aided max-SIR-link relay selection for an underlay CRN, in the presence of both primary source and destination. In the proposed scheme, the best relay corresponds to the highest SIR among all available source-to-relay and relay-to-destination links while keeping the interference at the primary user within a pre-defined level. The closed-form expression of the outage probability of the proposed scheme was obtained, which matches exactly the simulation results. Both theoretical and simulation results showed that the proposed scheme has significantly better outage performance than the conventional max-min scheme, making it an attractive scheme in the CRN.

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